

BT-102 (CBGS) B.Tech.,  
I & II Semester  
Examination, November 2019  
Choice Based Grading System (CBGS)  
Mathematics-I

Time : Three Hours

Maximum Marks : 70

- Note: i) Attempt any five questions.  
ii) All questions carry equal marks.  
iii) In case of any doubt or dispute the English version question should be treated as final.

1. a) Verify Rolle's theorem for the function  $f(x) = x^2 - x - 12$  in the interval  $[-3, 4]$ .  
b) Expand  $\log x$  in power of  $(x - 1)$  by Taylor's theorem and hence find the value of  $\log 1.1$ .

If  $u = \sin^{-1} \left( \frac{x^2 + y^2}{x + y} \right)$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$

2. a)

- b) Discuss the maximum or minima of the function  
 $f(x, y) = x^3 - 4xy + 2y^2$

3. a) Evaluate by expressing the following limit of a sum in the form of a definite integral.

$$\lim_{n \rightarrow \infty} \left\{ \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right\}$$

b)

Prove that  $\int_0^{\infty} \frac{x^c}{c^x} dx = \frac{\sqrt{c+1}}{(\log c)^{c+1}}$

(Answer)

4. a) Evaluate  $\iint_D x^2 y^2 dx dy$ , Where D is the region bounded by  $x = 0$ ,  $y = 0$  and  $x^2 + y^2 = 1$ ,  $x \geq 0$ ,  $y \geq 0$  \_\_\_\_\_

- b) Change the order of integration and evaluate it \_\_\_\_\_

$$\int_0^{\infty} \int_0^x e^{-xy} y dy dx.$$

5. a) Find the Fourier series for the function \_\_\_\_\_

$$f(x) = x^2, -\pi \leq x \leq \pi$$

b) Test the convergence of the series \_\_\_\_\_

$$\sqrt{\frac{1}{2^3}} + \sqrt{\frac{2}{3^3}} + \sqrt{\frac{3}{4^3}} + \sqrt{\frac{4}{5^3}} + \dots$$

6. a) Determine whether or not the vectors  $u(1, 1, 2)$ ,  $V(2,3,1)$ ,  $W(4,5,5)$  in  $R^3$  are linearly dependent.

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b) Let  $V=R^3$ , show that  $w$  is not a subspace of  $V$ , where  $w=\{(a,b,c): a \geq 0\}$

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7. a) Find the Rank of the matrix : \_\_\_\_\_

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

b) Find the characteristic equation of the matrix  $A$  and hence Find  $A^{-1}$ .

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

8. a) Find the Eigen values and Eigen vectors for the matrix  $A$ :

b) Show that the following equations are consistent or not.

$$\begin{aligned} 5x + 3y + 14z &= 4, \\ y + 2z &= 1, \\ x - y + 2z &= 0 \end{aligned}$$

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